

2nd derivative

Determine the 2nd derivative of the function defined implicitly

$$K) 2x^3 - 3y^2 = 8$$

$$\frac{dy}{dx} = \frac{x^2}{y}$$

$$y' = \frac{x^2}{y}$$

$$\frac{d^2y}{dx^2} =$$

$$y'' =$$

$$6x^2 - 6y \frac{dy}{dx} = 0$$

$$6x^2 = 6y \frac{dy}{dx}$$

$$\frac{6x^2}{6y} = \frac{dy}{dx}$$

$$\boxed{\frac{x^2}{y} = \frac{dy}{dx}}$$

$$L) x^{\frac{1}{3}} - y^{\frac{1}{3}} = 1$$

$$\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{3}y^{-\frac{2}{3}} \frac{dy}{dx} = 0$$

$$\frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3}y^{-\frac{2}{3}} \frac{dy}{dx}$$

$$\frac{\frac{1}{3}x^{-\frac{2}{3}}}{\frac{1}{3}y^{-\frac{2}{3}}} = \frac{dy}{dx}$$

$$\frac{x^{-\frac{2}{3}}}{y^{-\frac{2}{3}}} = \frac{dy}{dx}$$

$$\boxed{\frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}} = \frac{dy}{dx}}$$

$$\frac{d^2y}{dx^2} = \frac{y(2x) - x^2 \left( \frac{dy}{dx} \right)}{y^2}$$

$$\boxed{\frac{2xy - x^2 \left( \frac{x^2}{y} \right)}{y^2}}$$

$$\frac{7y^2 xy - x^4(y)}{y^2(y)}$$

$$\boxed{\frac{2xy^2 - x^4}{y^3}}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\frac{2}{3}x^{\frac{1}{3}} - \frac{2}{3}y^{-\frac{2}{3}} \frac{dy}{dx}}{(x^{\frac{1}{3}})^2} - y^{\frac{2}{3}} \left( \frac{2}{3}x^{-\frac{4}{3}} \right) \\ &= \frac{\frac{2}{3}x^{\frac{1}{3}} \left( y^{\frac{2}{3}} \right)}{x^{\frac{2}{3}}} - \frac{2y^{\frac{2}{3}}}{3x^{\frac{1}{3}}} \end{aligned}$$

$$\frac{\cancel{(x^{\frac{1}{3}})} \cancel{2y^{\frac{1}{3}}}}{\cancel{x}} - \frac{\cancel{2y^{\frac{2}{3}}}(3x^{\frac{1}{3}})}{\cancel{x^{\frac{2}{3}}}(3x^{\frac{1}{3}})}$$

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$$\boxed{\frac{2y^{\frac{1}{3}}(x^{\frac{1}{3}} - y^{\frac{1}{3}})}{3x^{\frac{5}{3}}}}$$

$$\boxed{\frac{2x^{\frac{1}{3}}y^{\frac{1}{3}} - 2y^{\frac{2}{3}}}{3x^{\frac{5}{3}}}}$$

$$\boxed{\frac{2\sqrt[3]{xy} - 2\sqrt[3]{y^2}}{3\sqrt[3]{x^5}}}$$

10. Consider the curve defined by the equation  $x^2 + xy + y^2 = 27$
- a) Write an expression for the slope of the curve at any point  $(x, y)$ .

$$2x + \left( x \frac{dy}{dx} + y \right) + 2y \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x+2y) = -2x-y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

$\frac{dy}{dx}$  und

- b) Find the points on the curve where the lines tangent to the curve are vertical.

$$x+2y=0$$

$$\begin{array}{l} x=-2y \\ y=3 \quad y=-3 \\ (-6, 3) \quad (6, -3) \end{array}$$

$$x^2 + xy + y^2 = 27$$

$$(-2y)^2 + y(-2y) + y^2 = 27$$

$$4y^2 - 2y^2 + y^2 = 27$$

$$\begin{aligned} 3y^2 &= 27 \\ y^2 &= 9 \end{aligned}$$

$$y = \pm 3$$

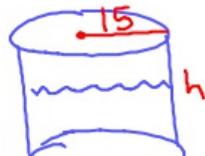
$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

- c) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(-2 - \frac{dy}{dx}\right) - (-2x-y)\left(1 + 2\frac{dy}{dx}\right)}{(x+2y)^2}$$

What you'll Learn About  
 How to use derivatives to solve a problem involving rates

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$$V = \pi r^2 h$$

- ① Draw a picture  
-only put constants
- ② Label all info
- ③ Find an eq. to take derivative of
- ④ Take Derivative

A) Water is draining from a cylindrical tank with radius of 15 cm at  $3000 \text{ cm}^3/\text{second}$ . How fast is the surface dropping?

$$\frac{dV}{dt} = -3000 \frac{\text{cm}^3}{\text{sec}}$$

$$\frac{dh}{dt} \frac{\text{cm}}{\text{sec}}$$

$$V = \pi r^2 h$$

$$V = \pi (15^2) h$$

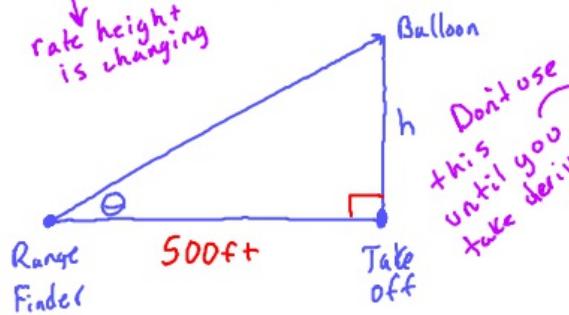
$$V = 225\pi h$$

$$-3000 = 225\pi \frac{dh}{dt}$$

$$\frac{-3000}{225\pi} = \frac{dh}{dt}$$

$$\frac{dV}{dt} = 225\pi \frac{dh}{dt}$$

B) A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is  $45^\circ$ , the angle is increasing at the rate of  $.14 \text{ rad/min}$ . How fast is the balloon rising at that moment?



At the moment

$$\theta = 45^\circ = \pi/4$$

$$\frac{d\theta}{dt} = .14 \frac{\text{radians}}{\text{min}}$$

Find  $\frac{dh}{dt}$

$$\tan \theta = \frac{h}{500} \quad h = 500 \tan \theta$$

$$\frac{dh}{dt} = 500 \sec^2 \theta \left( \frac{d\theta}{dt} \right)$$

$$\frac{dh}{dt} = 500 (\sec \frac{\pi}{4})^2 (.14)$$

$$\frac{dh}{dt} = 500 \left( \frac{2}{\sqrt{2}} \right)^2 (.14) \text{ ft/min}$$